On the robustness of the principal volatility components

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Abstract

In this paper, we analyse the recent principal volatility components analysis procedure. The procedure overcomes several difficulties in modelling and forecasting the conditional covariance matrix in large dimensions arising from the curse of dimensionality. We show that outliers have a devastating effect on the construction of the principal volatility components and on the forecast of the conditional covariance matrix and consequently in economic and financial applications based on this forecast. We propose a robust procedure and analyse its finite sample properties by means of Monte Carlo experiments and also illustrate it using empirical data. The robust procedure outperforms the classical method in simulated and empirical data.

Keywords: Conditional covariance matrix; Constant volatility; Curse of dimensionality; Jumps; Outliers; Principal components.

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1 Introduction

Modelling and forecasting volatilities and co-volatilities play a crucial role in many economic and financial applications such as portfolio allocation, risk measures, option pricing, secu-

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rity regulations and hedging strategies (Chiou and Tsay, 2008; Hammoudeh et al., 2010; Rombouts and Stentoft, 2011; Basher and Sadorsky, 2016; Wang and Liu, 2016).

Given the infeasibility and inflexibility of most classical multivariate volatility models in large dimensions, researchers and practitioners have been looking for alternative tools to circumvent the curse of dimensionality when modelling and forecasting (co)volatilities in high-dimensional data. In this sense, some alternative approaches have been suggested in the last years. See, for instance, Lopes et al. (2012), Fan et al. (2012), Hafner and Reznikova (2012), Pakel et al. (2014), Gruber and West (2016), Kastner (2016), Li et al. (2016) and Engle et al. (2017), among others. Furthermore, based on the idea that co-movements in the market can be driven by a few components, factor models appear in the economic and financial literature as an alternative way to achieve dimension reduction and to tackle the curse of dimensionality. See, for instance, Fan et al. (2008), Pan et al. (2010), Matteson and Tsay (2011), García-Ferrer et al. (2012), Santos and Moura (2014), Matilainen et al. (2015) and Barigozzi and Hallin (2015) for some references.

In the spirit of dimensionality reduction, an innovative approach based on the classical principal component analysis (PCA), called principal volatility components (PVC), has been recently proposed by Hu and Tsay (2014a) and Li et al. (2016). This methodology produces two types of components. The first type corresponds to components with conditional covariance matrix evolving over time whilst the other type corresponds to components with constant conditional covariance matrix. This methodology is attractive because after obtaining the volatility components, the problem of modelling and forecasting the (co)volatilities of the entire system drop down into modelling and forecasting the (co)volatilities of the volatility components with heteroscedastic dynamics since the remaining components have constant volatility.

On the other hand, it is well known that outliers are not unusual in financial time series and several works show how outliers affect dramatically the forecast of (co)volatilities (Muler and Yohai, 2008; Boudt and Croux, 2010; Carnero et al., 2012; Boudt et al., 2013; Grané et al., 2014; Trucíos and Hotta, 2016; Trucíos et al., 2017; Trucíos, 2018) and consequently financial applications (Mendes and Leal, 2005; Welsch and Zhou, 2007; Trucíos et al., 2018). See, Hotta and Trucíos (2018) for a good recent review about outliers in

(M)GARCH models. According to Sakata and White (1998) , "A prominent characteristic of asset price movements is the episodic occurrence of crashes and rallies,.." which will lead to outliers. For instance, an abnormal price in an isolated day or an sudden increase or decrease of an asset price lead to additive type outliers.

Furthermore, there is evidence showing that PCA is very sensitive to the presence of outliers (Croux and Haesbroeck, 2000; Hubert et al., 2005; Candès et al., 2011; Greco and Farcomeni, 2016). Thus, procedures based on similar methodology are also expected to be sensitive to outliers.

The contribution of this paper is threefold. First, by means of Monte Carlo experiments, we investigate the performance of PVC in the presence of additive outliers showing that they have a devastating effect on this procedure, even when moderate outliers are present. Second, we propose a robust principal volatility component (RPVC) procedure which shows to have good finite sample properties and also does not suffer the Lucas critique [1]. Third, we compare empirically the GPVC and RPVC procedures in a Value-at-Risk and minimum variance portfolio context showing that a better portfolio performance can be obtained using the latter procedure.

The rest of the paper is organized as follows. Section 2 presents the PVC of Hu and Tsay (2014a), the generalized version of Li et al. (2016) and our robust procedure. In Section 3 an extensive Monte Carlo experiment is carried out to evaluate the finite sample properties of the procedures in contaminated and uncontaminated series. Section 4 presents an empirical application of daily returns with 73 stocks of the Nasdaq-100 index and show that our robust procedure has better performance when applied to the selection of the minimum variance portfolio. Finally, Section 5 presents the main conclusions and future works.

[1]Lucas critique is a very important concept in economics and also in econometrics. The super-exogeneity test proposed by Favero and Hendry (1992) is one way to test Lucas critique. Hendry and Santos (2010) shows the relationship between outliers and structural break with super-exogeneity and they show that outliers and structural breaks can induce loss of invariance, therefore Lucas critique, even when the model is invariant. These procedures are valid for models for the conditional mean and also for the conditional variance. Our method that robustify the PVC methods does not suffer from the Lucas critique which is not the case for the usual methods of PVC.

2 Volatility components

Let $y_t = (y_{1t}, ..., y_{Nt})'$ a N-dimensional vector with $E(y_t|\mathcal{F}_{t-1}) = 0$ where $\mathcal{F}_{t-1}$ denotes the information available up to time $(t-1)$ and let $M_{N\times N} = [A_{N\times r} \quad B_{N\times(N-r)}]$ an orthogonal matrix. Observe that if we denote $f_t = A'y_t$ and $\epsilon_t = BB'y_t$, we can rewrite y_t as

$$y_t = MM'y_t = (AA' + BB')y_t = Af_t + \epsilon_t. \tag{1}$$

Hu and Tsay (2014a) and Li et al. (2016) introduce methodologies on which, under mild conditions, it is possible to find B such that $Var(\epsilon_t|\mathcal{F}_{t-1}) = Var(\epsilon_t)$, i.e. , the second term ϵ_t contains homoscedastic components and all the conditional heteroscedastic components come from the first term. Although model (1) has the same form of the classical factor model, there are some differences between them. First, model (1) splits y_t into two terms, one explaining the conditional heteroscedastic dynamic (f_t) and the other one driven by components with constant volatility (ϵ_t). Additionally, f_t and ϵ_t are not necessary uncorrelated. Finally, none assumption is imposed directly on f_t and ϵ_t and all the features described previously are consequences of the eigenvalue-eigenvector decomposition described in Hu and Tsay (2014a) and Li et al. (2016) respectively. This model is also particularly useful because it reduces considerably the number of parameters to be estimated circumventing the curse of dimensionality.

We briefly introduce the approaches of Hu and Tsay (2014a) and Li et al. (2016), denoted by PVC and GPVC respectively. These approaches allow obtaining components with the features described previously. Additionally, knowing the bad influence of outliers in classical methodologies and inspired on the comments of Franke (2014) and Hu and Tsay (2014b) about the robustness of the PVC procedure, we introduce a robust procedure which is less sensitive to additive outliers.

2.1 Principal volatility components (PVC)

Let us assume that the vector y_t defined previously is weakly stationary with finite fourth-order moment. Hu and Tsay (2014a) consider the eigenvalue-eigenvector decomposition of the cumulative generalized kurtosis matrix given by $\Gamma_\infty M = \Lambda M$ where Λ is a diagonal matrix of eigenvalues in descending order, M is the associated normalized eigenvectors and Γ_∞ is the cumulative generalized kurtosis matrix of y_t defined as

$$\Gamma_\infty = \sum_{k=1}^{\infty}\sum_{i=1}^{N}\sum_{j=1}^{N} E^2\left[\left(y_t y_t' - \Sigma\right)\left(x_{ij,t-k} - E\left(X_{ij}\right)\right)\right], \tag{2}$$

where Σ is the unconditional covariance matrix and $x_{ij,t-k} = r(y_{i,t-k}y_{j,t-k})$, i.e., it is a function of the cross-product $y_{i,t-k}y_{j,t-k}$[2]. The k-th volatility component is defined as $z_{kt} = m_k' y_t$, where m_k is the eigenvector associated with the k-th largest eigenvalue and corresponds to the k-th column of M. Hu and Tsay (2014a) proves that if m_k is an eigenvector associated with a zero eigenvalue of Γ_∞, the linear combination $m_k' y_t$ has constant volatility (See Lemma 1 — Theorem 1 of Hu and Tsay (2014a)). Additionally, it can also be proved that under mild conditions (Theorem 1 of Hu and Tsay (2014a)) there exists $N-r$ linearly independent combinations of y_t with constant volatility, where $r = rank(\Gamma_\infty)$.

In practice, (2) is estimated by

$$\hat{\Gamma}_g = \sum_{k=1}^{g}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(1-\frac{k}{T}\right)^2\left[\frac{1}{T}\sum_{t=k+1}^{T}\left[\left(y_t y_t' - \hat{\Sigma}\right)\left(x_{ij,t-k} - \bar{x}_{ij}\right)\right]\right]^2, \tag{3}$$

where $\hat{\Sigma}$ is the sample covariance matrix, $\bar{x}_{ij}$ is the sample mean of $x_{ij,t}$, g is a positive integer that represents a lag order and T is the sample size. For more details see Hu and Tsay (2014a) and Andreou and Ghysels (2014).

As pointed out by Hu and Tsay (2014a), traditional PC methods applied to y_t depends on the covariance matrix, while PVC is focused on the dynamic dependence of the volatility and concerned with the fourth moments.

2.2 Generalized principal volatility components (GPVC)

The PVC of Hu and Tsay (2014a) assumes that the vector series has finite fourth-order moment. However, there is evidence showing that in many financial series this assumption

[2] In their simulations and empirical application Hu and Tsay (2014a) use the Huber's function defined as

$$r(w) = \begin{cases} w, & \text{if } |w| \le c^2, \\ 2c\sqrt{w} - c^2, & \text{if } w > c^2, \\ c^2 - 2c\sqrt{|w|}, & \text{if } w < -c^2. \end{cases}$$

does not hold (Zhu and Ling, 2011). To relax this assumption, Li et al. (2016), inspired by the paper of Pan et al. (2010), propose an alternative PVC procedure, denoted by GPVC, which requires only finite second-order moments.

In the GPVC, the cumulative generalized kurtosis matrix (2) is replaced by

$$G = \sum_{k=1}^{g} \sum_{t=1}^{T} \omega(y_t) E^2 \left[(y_t y_t' - \Sigma) I(\|y_{t-k}\| \leq \|y_t\|) \right], \tag{4}$$

where g is a fixed number, $\omega(\cdot)$ is a weight function and $\|\cdot\|$ is the L_1 norm. Li et al. (2016) test different values of g and conclude that the procedure is robust to the choice of g. The matrix G is estimated in a natural way by

$$\hat{G} = \sum_{k=1}^{g} \sum_{\tau=1}^{T} \omega(y_\tau) \left[\frac{1}{T-k} \sum_{t=k+1}^{T} \left[\left(y_t y_t' - \hat{\Sigma} \right) I(\|y_{t-k}\| \leq \|y_\tau\|) \right] \right]^2. \tag{5}$$

Similarly to the PVC procedure, the GPVC procedure considers the k-th volatility component estimated as $z_{kt} = m_k' y_t$, but with $\hat{\Gamma}_g$ substituted by $\hat{G}$. An important difference between PVC and GPVC is that the first is based on the fourth moment, while the second is based on second moments.

Both procedures present a good performance with a slightly better performance in favour of the GPVC procedure, mainly when ϵ_t is heavy-tailed distributed

However, these procedures have two drawbacks. The first one, which is not discussed here, is related to the problem of dealing with $N/T \to 1$ or even $N > T$. The second one, which is the focus of this paper, is related to the presence of additive outliers that, as discussed previously, can have several implications in modelling and forecasting volatility (Boudt et al., 2013; Grané et al., 2014; Trucíos et al., 2017, 2018). These outliers are not unusual and can be related to financial crashes, elections, wars, macroeconomic news and terrorist attack (Charles and Darné, 2014; Laurent et al., 2016).

Because both procedures are based on a methodology similar to the classical PCA, which is very sensitive to atypical observations (Croux and Haesbroeck, 2000; Hubert et al., 2005; Candès et al., 2011; Greco and Farcomeni, 2016) and in addition considering that both procedures focus on the estimation and prediction of the conditional covariance matrix, which are badly affected by additive outliers (Carnero et al., 2012; Boudt et al., 2013;

Trucíos et al., 2017, 2018) it is important to know whether and how outliers affect the (G)PVC procedures and consequently their financial applications. In a second step, it is interesting to find an alternative or a robust procedure, which is pursued in the following section.

2.3 Robust principal volatility components (RPVC)

In order to obtain a procedure less sensitive to additive outliers, we robustify the estimator given in (5). The robust procedure is based on a robust estimator of the unconditional covariance matrix and a weighted estimator of $E\left[(y_t y_t' - \Sigma)\, I(\|y_{t-k}\| \leq \|y_t\|)\right]$. We replace the matrix (5) by a less sensitive matrix defined as

$$\hat{G}^R = \sum_{k=1}^{g} \sum_{\tau=1}^{T} \omega(y_\tau) \left[\sum_{t=k+1}^{T} \delta^*(d_t^2) \left\{ (y_t y_t' - \hat{\Sigma}^R) I(\|y_{t-k}\| \leq \|y_\tau\|) \right\} \right]^2, \tag{6}$$

where $\omega(\cdot) = 1/T$ as in Li et al. (2016)[3], d_t^2 is the robust square Mahalanobis distance given by $d_t^2 = (y_t - \hat{\mu}^R)' \hat{\Sigma}_t^{-1} (y_t - \hat{\mu}^R)$ with $\hat{\Sigma}_t = 0.01\rho(y_{t-1}' y_{t-1}) + 0.99\hat{\Sigma}_{t-1}$, $\hat{\Sigma}_1 = \hat{\Sigma}^R$ and $\hat{\mu}^R$ and $\hat{\Sigma}^R$ being a robust estimates of the unconditional mean and covariance matrix obtained using the minimum covariance determinant (MCD) estimator of Rousseeuw (1984) implemented with the algorithm of Hubert et al. (2012). Finally, $\rho(\cdot)$ and $\delta(\cdot)$ are given by

$$\rho(x_t) = \begin{cases} x_t, & \text{if } d_t^2 \leq c, \\ \hat{\Sigma}^R, & \text{if } d_t^2 > c, \end{cases} \qquad \delta(x) = \begin{cases} 1, & \text{if } x \leq c, \\ \frac{1}{x}, & \text{if } x > c, \end{cases}$$

and $\delta^*(\cdot) = \delta(\cdot)/||\delta(\cdot)||$, where $\|\cdot\|$ is the L_1 norm. The value c in $\rho(\cdot)$ and $\delta(\cdot)$ is a threshold parameter defined prior to the estimation procedure. In our simulations and empirical application we use c as the 0.99 quantile of the empirical distribution of d_t^2.

Observe that, to avoid that returns corresponding to periods with high volatility being considered as possible outliers we incorporate in the squared Mahalanobis distance a covariance matrix evolving obtained by a procedure that can be seen as a robust RiskMetrics 1994 Smoother with $\lambda = 0.99$. Similar approaches have also been used in, for instance, Boudt and Croux (2010), Croux et al. (2010) and Boudt et al. (2013). Additionally, because the sample covariance matrix is sensitive to outliers (Hubert et al., 2012, 2015), we use the robust MCD estimator (Rousseeuw, 1984; Hubert et al., 2012). To maintain the robustness of d_t^2, we use $\hat{\Sigma}^R$ as the initial value in $\hat{\Sigma}_t$ and introduce the filter $\rho(\cdot)$ that mitigate the

[3]We have also tried different weight functions but none of them robustify the GPVC procedure.

effect of outliers in the RiskMetrics Smoother. Finally, as a natural robust estimator of $E\left[(y_t y_t' - \Sigma)\, I(\|y_{t-k}\| \leq \|y_t\|)\right]$ we use a weighted estimator that penalizes large values of d_t^2.[4] Note that the modifications mitigate the influence of the additive type of outliers.

3 Monte Carlo experiments

To evaluate the finite sample properties of the PVC, GPVC and RPVC, we carry out Monte Carlo experiments with small and large dimensions. Following a value frequently used in the financial literature (McNeil and Frey, 2000; Sentana et al., 2008; Matteson and Tsay, 2011; Luciani and Veredas, 2015; Engle et al., 2017; Trucíos et al., 2018) we consider series of sample size 1000, which is approximately 4 years of daily data.

Different patterns of contamination, size of outliers and percentage of series contaminated are considered. We consider consecutive (C) and isolated (I) outliers in the middle and close to the end of the sample period. In cases contaminated by isolate outliers, we added two outliers in the series at positions $t = 500$ and 999. In a similar way, we added outliers at positions $t = 500, 501$ and $998, 999$ when two consecutive outliers are considered. Outliers of size 5 and 10 standard deviations of the univariate uncontaminated process are contemplated. Finally, we consider uncontaminated series (0% of series contaminated) and contamination of $p\%$ of the series, with $p\% = 25\%, 50\%$ and 100% of the series. For the $p\%$ contaminated series we contaminated the first $p\%$ series which appear in the entire system.

Following Andreou and Ghysels (2014), Hu and Tsay (2014a) and Li et al. (2016), we use the factor model as data generating process (DGP). In the simulation study, we consider three analyses. First, we analyze if outliers affect the estimation of the number of volatility components with heteroskedastic dynamic. Second, considering that the number

[4] At the same time we are working in a robust PVC procedure, another robust procedure is being developed independently by Monte and Reisen (2016). The main differences between both approaches are that Monte and Reisen (2016) robustify the procedure of Hu and Tsay (2014a) while we robustify the procedure of Li et al. (2016). The procedure of Monte and Reisen (2016) replace the generalised covariance in (3) by a robust version based on Ma and Genton (2000) while we use the robust MCD estimator of Rousseeuw (1984). Additionally, we mitigate the effect of outliers penalizing large values of the squared Mahalanobis distance taking into account high and low volatility period using a RiskMetrics Smoother that avoid returns corresponding to periods with high volatility being considered as outliers. Finally, the robust procedure proposed in this paper is fast and feasible in large dimensions. Because the GPVC has shown a slight better performance than the PVC and in addition, because the robust procedure of Monte and Reisen (2016) is computationally more expensive than the other approaches we do not analyse this procedure here since Monte Carlo experiments even for small dimension ($N = 8$) is highly time consuming and in consequence infeasible in moderate/large dimensions.

of components with heteroskedastic dynamic is known, we analyze the effect of the outliers in the estimation of the matrix A in (1). Finally, we are interested in the effects of outliers in the prediction of the conditional covariance matrix and its implications in economic and financial applications. In the simulation and in the application we use maximum lag g equal to 5 in estimators (5) and (6).

3.1 Number of volatility components

First, we are interested in whether the selection of the number of volatility components with heteroskedastic dynamic is affected by outliers. It is important to mention that, as far as we know, there is no optimal procedure developed to select the optimal number of volatility components. For simplicity and illustrative purposes, we use three criteria commonly used in principal components and factor analysis context. Specifically, we use the ratio estimator criterion (Lam and Yao, 2012; Ahn and Horenstein, 2013), the BN criterion (Bai and Ng, 2002) and the Kaiser-Guttman criterion (Guttman, 1954).

We consider small and large dimensions ($N = 8$ and 60) and the factor model was generated with two and six common factors for $N = 8$ and 60 respectively. Each common factor follows a Gaussian GARCH(1,1) process with parameters $\omega = (1, 2)$, $\alpha = (0.07, 0.03)$ and $\beta = (0.83, 0.92)$ for two factors and $\omega = (1, 2, 1, 0.5, 2, 3)$, $\alpha = (0.07, 0.03, 0.05, 0.03, 0.02, 0.03)$ and $\beta = (0.83, 0.92, 0.90, 0.95, 0.78, 0.87)$ for six factors. The factor load matrix A was randomly drawn as a matrix with orthogonal columns using the R package *pracma* of Borchers (2017) and the idiosyncratic factors were simulated as $\epsilon_t = \frac{\bar{\epsilon}_t}{\sqrt{N}}$ where $\bar{\epsilon}_t \sim Normal_N(0, I_N)$ with N being the dimension of the system and I_N the identity matrix of order $N \times N$. The initial covariance matrix H_0 was simulated as a positive definite matrix using the procedure of Joe (2006) implemented in R package *clusterGeneration* of Qiu and Joe. (2015). In all cases we simulate 1500 observations and discard the first 500 to avoid the influence of the initial values.

Tables 1 and 2 report the average and standard deviation of the estimated number of volatility components using the ratio estimator (top panel), the BN[5] (middle panel) and the Kaiser-Guttman (bottom panel) criteria for small and large dimensions respectively.

[5]Following the rule used in Bai and Ng (2002) the maximum number of components are 4 and 8 for dimension $N = 8$ and 60 respectively.

Additionally, the proportion of estimated components smaller, equal to and larger than the true number of factors are also reported. Observe that, mainly for the large dimension case, when the non-robust procedures are implemented, the selected number of estimated volatility components obtained using any of the three criteria mentioned previously is highly affected by the presence of outliers. For large dimensions, when the RPVC procedure is used all criteria estimate correctly the number of components in the presence and absence of outliers. In small dimension, the ratio estimator and BN criteria estimated correctly the number of volatility components most of times while the Kaiser-Guttman criterion underestimated the number of components more than 40% of times.

In the presence of outliers, when the non-robust procedures are used, the BN criterion overestimates the number of components in both small and large dimension. In large dimensions, the ratio estimator criterion also overestimates the number of volatility components, with an exception observed when the PVC procedure is used in a context of consecutive outliers in 100% of series, in which case the number of selected volatility components is close the obtained using the Kaisser-Gutman criterion. In small dimensions, the ratio estimator criterion misestimates the number of selected volatility components in some cases, being the overestimation case the more frequent. The Kaisser-Guttman criterion underestimates the number of components in both small and large dimension. Note also that in large dimension and absence of outliers, the Kaisser-Guttman criterion estimates correctly the number of components but in small dimension underestimate the components. Additional Monte Carlo experiments conclude that as the ratio *common factors/dimension* increase, the Kaiser-Guttman criterion estimate incorrectly the number of components (see supplementary material).

It is important to point out that additional Monte Carlo experiments to analyze deeply in which cases the ratio estimator and the Kaisser-Guttman criteria can either overestimate or underestimate the number of components is necessary to be made. However, this deserves an additional study. We can conclude that in the presence of outliers the criteria estimate incorrectly the number of components when the DGP is the factor model.

Table 1: Average and standard deviation of the estimated number of components for uncontaminated and contaminated series using the ratio estimator (top panel), the BN (middle panel) and the Kaiser-Guttman (bottom panel) criteria. Rows (Comp = 1), (Comp =2) and (Comp >2) present the proportion of cases where the estimated number of components are smaller, equal and larger than the number of true factors, respectively. Dimension $N = 8$, sample size $T = 1000$ and 1000 Monte Carlo replicates. The factor models are simulated with two factors.

	(a)	0%	25%				50%				100%			
	(b)		$\omega = 5$		$\omega = 10$		$\omega = 5$		$\omega = 10$		$\omega = 5$		$\omega = 10$	
	(c)		I	C	I	C	I	C	I	C	I	C	I	C
Ratio estimator criterion														
RPVC	Mean	1.992	1.986	1.988	1.987	1.990	1.988	1.989	1.988	1.987	1.987	1.988	1.987	1.988
	SD	0.089	0.118	0.109	0.113	0.100	0.109	0.104	0.109	0.113	0.113	0.109	0.113	0.109
	Comp =1	0.008	0.014	0.012	0.013	0.010	0.012	0.011	0.012	0.013	0.013	0.012	0.013	0.012
	Comp =2	0.992	0.986	0.988	0.987	0.990	0.988	0.989	0.988	0.987	0.987	0.988	0.987	0.988
	Comp >2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GPVC	Mean	1.998	2.383	2.607	3.106	3.366	2.868	3.100	3.249	3.798	2.281	2.624	2.032	3.108
	SD	0.045	0.830	1.017	0.737	0.864	1.031	1.465	0.952	1.385	0.957	1.370	0.830	1.176
	Comp =1	0.002	0.151	0.173	0.050	0.068	0.152	0.233	0.100	0.126	0.222	0.245	0.265	0.106
	Comp =2	0.998	0.390	0.271	0.074	0.053	0.150	0.115	0.059	0.060	0.409	0.302	0.502	0.193
	Comp >2	0.000	0.459	0.556	0.876	0.879	0.698	0.652	0.841	0.814	0.369	0.453	0.233	0.701
PVC	Mean	1.972	2.214	2.694	3.037	2.779	2.517	2.944	3.286	2.437	2.090	1.880	2.143	1.987
	SD	0.165	0.794	1.012	0.785	1.268	1.065	1.523	0.908	1.626	0.878	0.975	0.960	0.886
	Comp =1	0.028	0.189	0.168	0.069	0.291	0.242	0.290	0.085	0.434	0.258	0.409	0.284	0.303
	Comp =2	0.972	0.450	0.207	0.083	0.065	0.199	0.110	0.055	0.216	0.480	0.407	0.404	0.497
	Comp >2	0.000	0.361	0.625	0.848	0.644	0.559	0.600	0.860	0.350	0.262	0.184	0.312	0.200
BN criterion														
RPVC	Mean	2.066	2.400	2.502	2.588	2.620	2.449	2.517	2.598	2.599	2.328	2.416	2.532	2.579
	SD	0.293	0.726	0.783	0.818	0.833	0.743	0.799	0.814	0.825	0.650	0.732	0.791	0.815
	Comp =1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp =2	0.946	0.743	0.679	0.625	0.609	0.703	0.677	0.613	0.621	0.773	0.730	0.656	0.631
	Comp >2	0.054	0.257	0.321	0.375	0.391	0.297	0.323	0.387	0.379	0.227	0.270	0.344	0.369
GPVC	Mean	2.013	3.352	3.650	3.422	3.704	3.658	3.899	3.691	3.872	3.765	3.888	3.856	3.823
	SD	0.137	0.531	0.498	0.498	0.457	0.477	0.308	0.462	0.346	0.447	0.334	0.354	0.460
	Comp =1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
	Comp =2	0.990	0.027	0.010	0.002	0.000	0.001	0.002	0.000	0.004	0.010	0.006	0.001	0.030
	Comp >2	0.010	0.973	0.990	0.998	1.000	0.999	0.998	1.000	0.996	0.990	0.994	0.999	0.969
PVC	Mean	2.026	3.348	3.651	3.438	3.701	3.649	3.901	3.690	3.891	3.696	3.924	3.845	3.888
	SD	0.208	0.551	0.497	0.500	0.458	0.480	0.299	0.463	0.324	0.508	0.283	0.376	0.337
	Comp =1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
	Comp =2	0.983	0.038	0.010	0.002	0.000	0.001	0.000	0.000	0.001	0.023	0.005	0.005	0.007
	Comp >2	0.017	0.962	0.990	0.998	1.000	0.999	1.000	1.000	0.998	0.977	0.995	0.995	0.993
Kaiser-Guttman criterion														
RPVC	Mean	1.562	1.549	1.522	1.552	1.521	1.550	1.520	1.548	1.518	1.544	1.517	1.546	1.518
	SD	0.496	0.498	0.500	0.498	0.500	0.498	0.500	0.498	0.500	0.498	0.500	0.498	0.500
	Comp =1	0.438	0.451	0.478	0.448	0.479	0.450	0.480	0.452	0.482	0.456	0.483	0.454	0.482
	Comp =2	0.562	0.549	0.522	0.552	0.521	0.550	0.520	0.548	0.518	0.544	0.517	0.546	0.518
	Comp >2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GPVC	Mean	1.494	1.527	1.556	1.741	1.658	1.581	1.568	1.661	1.601	1.588	1.599	1.549	1.679
	SD	0.500	0.500	0.503	0.533	0.543	0.494	0.500	0.578	0.551	0.499	0.502	0.498	0.490
	Comp =1	0.506	0.473	0.447	0.305	0.377	0.419	0.434	0.394	0.431	0.415	0.407	0.451	0.332
	Comp =2	0.494	0.527	0.550	0.649	0.588	0.581	0.564	0.551	0.537	0.582	0.587	0.549	0.657
	Comp >2	0.000	0.000	0.003	0.046	0.035	0.000	0.002	0.055	0.032	0.003	0.006	0.000	0.011
PVC	Mean	1.433	1.467	1.508	1.674	1.422	1.496	1.478	1.634	1.324	1.528	1.382	1.465	1.391
	SD	0.496	0.503	0.512	0.512	0.548	0.502	0.516	0.541	0.475	0.503	0.488	0.501	0.488
	Comp =1	0.567	0.535	0.498	0.347	0.606	0.505	0.530	0.396	0.679	0.474	0.619	0.536	0.609
	Comp =2	0.433	0.463	0.496	0.632	0.366	0.494	0.462	0.574	0.318	0.524	0.380	0.463	0.391
	Comp >2	0.000	0.002	0.006	0.021	0.028	0.001	0.008	0.030	0.003	0.002	0.001	0.001	0.000

(a) percentage of series contaminated. (b) Size of outliers in terms of standard deviations of the univariate uncontaminated process. (c) position of outliers: at time $t = 500$ and 999, (I)solated outliers or at times $t = 500, 501$ and $998, 999$, (C)onsecutive outliers.

Table 2: Average and standard deviation of the estimated number of components for uncontaminated and contaminated series using the ratio estimator (top panel), the BN (middle panel) and the Kaiser-Guttman (bottom panel) criteria. Rows (Comp < 6), (Comp $=6$) and (Comp >6) present the proportion of cases where the estimated number of components are smaller, equal and larger than the number of true factors, respectively. Dimension $N = 60$, sample size $T = 1000$ and 1000 Monte Carlo replicates. The factor models are simulated with six factors.

	(a)	0%	25%				50%				100%			
	(b)		$\omega = 5$		$\omega = 10$		$\omega = 5$		$\omega = 10$		$\omega = 5$		$\omega = 10$	
	(c)		I	C	I	C	I	C	I	C	I	C	I	C
						Ratio estimator criterion								
RPVC	Mean	6.000	6.000	6.000	6.000	6.000	6.000	6.000	6.000	6.000	6.000	6.000	6.000	6.000
	SD	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp <6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp $=6$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	Comp >6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GPVC	Mean	6.000	7.980	9.926	7.990	9.960	7.999	9.997	7.999	9.997	8.000	10.000	7.766	9.604
	SD	0.000	0.140	0.298	0.100	0.196	0.032	0.055	0.032	0.055	0.000	0.000	1.162	1.490
	Comp <6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.039	0.066
	Comp $=6$	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp >6	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.961	0.934
PVC	Mean	6.000	7.976	9.941	7.990	9.927	7.999	9.997	7.999	8.886	8.000	6.647	7.832	2.050
	SD	0.000	0.153	0.252	0.100	0.597	0.032	0.055	0.032	2.769	0.000	3.950	0.990	0.621
	Comp <6	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.139	0.000	0.419	0.028	0.994
	Comp $=6$	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp >6	0.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000	0.861	1.000	0.581	0.972	0.006
						BN criterion								
RPVC	Mean	6.001	6.005	6.005	6.023	6.008	6.005	6.004	6.017	6.010	6.008	6.002	6.013	6.003
	SD	0.032	0.071	0.071	0.157	0.089	0.071	0.063	0.144	0.100	0.089	0.045	0.122	0.055
	Comp <6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp $=6$	0.999	0.995	0.995	0.978	0.992	0.995	0.996	0.985	0.990	0.992	0.998	0.988	0.997
	Comp >6	0.001	0.005	0.005	0.022	0.008	0.005	0.004	0.015	0.010	0.008	0.002	0.012	0.003
GPVC	Mean	6.000	7.986	8.000	7.991	7.999	7.999	7.999	7.999	8.000	8.000	8.000	8.000	8.000
	SD	0.000	0.118	0.000	0.094	0.032	0.032	0.032	0.032	0.000	0.000	0.000	0.000	0.000
	Comp <6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp $=6$	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp >6	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
PVC	Mean	6.000	7.987	8.000	7.990	8.000	7.999	8.000	7.999	7.999	8.000	8.000	8.000	8.000
	SD	0.000	0.113	0.000	0.100	0.000	0.032	0.000	0.032	0.032	0.000	0.000	0.000	0.000
	Comp <6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp $=6$	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Comp >6	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
						Kaiser-Guttman criterion								
RPVC	Mean	5.999	5.999	6.000	5.999	6.000	6.000	6.000	6.000	6.000	6.000	5.998	6.000	6.000
	SD	0.032	0.032	0.000	0.032	0.000	0.000	0.000	0.000	0.000	0.000	0.045	0.000	0.000
	Comp <6	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000
	Comp $=6$	0.999	0.999	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000
	Comp >6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GPVC	Mean	6.000	7.218	7.868	7.341	7.453	7.574	8.008	4.901	4.654	5.963	5.701	2.183	3.995
	SD	0.000	0.534	0.630	0.660	0.854	0.532	0.737	0.765	0.677	0.508	0.636	0.389	0.071
	Comp <6	0.000	0.000	0.000	0.003	0.014	0.000	0.000	0.787	0.904	0.144	0.353	1.000	1.000
	Comp $=6$	1.000	0.057	0.007	0.096	0.115	0.019	0.017	0.209	0.091	0.747	0.572	0.000	0.000
	Comp >6	0.000	0.943	0.993	0.901	0.871	0.981	0.983	0.004	0.005	0.109	0.075	0.000	0.000
PVC	Mean	6.000	7.088	7.968	7.617	3.315	7.599	5.089	5.588	2.117	6.188	2.294	3.288	2.000
	SD	0.000	0.568	0.501	0.615	0.783	0.502	0.832	0.995	0.322	0.539	0.458	0.860	0.000
	Comp <6	0.000	0.000	0.000	0.000	0.989	0.000	0.690	0.429	1.000	0.062	1.000	0.996	1.000
	Comp $=6$	1.000	0.121	0.004	0.071	0.010	0.006	0.277	0.424	0.000	0.686	0.000	0.004	0.000
	Comp >6	0.000	0.879	0.996	0.929	0.001	0.994	0.033	0.147	0.000	0.252	0.000	0.000	0.000

(a) percentage of series contaminated. (b) Size of outliers in terms of standard deviations of the univariate uncontaminated process. (c) position of outliers: at time $t = 500$ and 999, (I)solated outliers or at times $t = 500, 501$ and $998, 999$, (C)onsecutive outliers.

3.2 Eigenvectors associated with non-zero eigenvalues

In this section, we analyze the effects of outliers on the estimation of the eigenvectors associated with the non-zero eigenvalues. Note that, these vectors are the columns of the matrix A in the model $y_t = Af_t + \epsilon_t$ and play an important role on the forecast of the conditional covariance matrix. To separate the source of error, we focus on the estimation of the eigenvectors and assume that we know the true number of components with heteroscedastic dynamics. We follow Li et al. (2016) and carry out a similar Monte Carlo experiments with 1000 replicates and consider small ($N = 8$) and large ($N = 100$) dimension cases. Following examples 1 and 4 of Li et al. (2016), the factor model is driven by just one common factor which follows a Gaussian GARCH(1,1) process with parameters $\omega = 1$, $\alpha = 0.07$ and $\beta = 0.83$. The idiosyncratic factors are generated as in Subsection 3.1. The factor load matrix A is also normalized and each element is a random draw of $U(-1, 1)$. Given that the PVC and GPVC procedures have similar performance (Li et al., 2016) and considering the extreme computational cost of the Monte Carlo experiment using the PVC procedure when $N = 100$, for large dimensions we only consider the GPVC and RPVC procedures. To compare the estimation of the matrix A we use the two measures[6] defined in Li et al. (2016) and given by

$$d\left(\hat{\mathcal{M}}_1, \mathcal{M}_1\right) = \sqrt{1 - \frac{Tr\left(\hat{A}\hat{A}'AA'\right)}{r}}, \tag{7}$$

$$d\left(\hat{A}, A\right) = 1 - \frac{\left[\sum_t (y_t - \bar{y})'\hat{A}A'(y_t - \bar{y})\right]^2}{\left[\sum_t (y_t - \bar{y})'\hat{A}\hat{A}'(y_t - \bar{y})\right]\left[\sum_t (y_t - \bar{y})'AA'(y_t - \bar{y})\right]}, \tag{8}$$

where y_t is a vector of observed returns, A is the true load factor matrix, $\hat{A}$ is the estimated load factor matrix, r is the number of columns of $\hat{A}$, and $\mathcal{M}_1$ ($\hat{\mathcal{M}}_1$) is the linear space spanned by the columns of A ($\hat{A}$), i. e., the volatility (estimated volatility) space. Figure 1 presents the box-plot of the results of the measure defined in (7) for small and large dimension cases. We can observe that the effect of outliers in PVC and GPVC procedures is devastating even when just a few outliers are added in the series.

In general, the analyses of the results show that the RPVC is less sensitive to outliers

[6]These measures and the measures used in the next section are implemented in the R package *StatPerMeco* of Trucíos (2017).

and also stable regardless the proportion of series contaminated, the size of outliers and whether the outliers are isolated or consecutive. Note also that, in the absence of outliers, the performance of all procedures is almost similar to a slightly better performance of the GPVC procedure. Results using (8) produces similar results and are available in the supplementary material.

3.3 Conditional covariance matrix

In this last section on Monte Carlo experiment, we use the same DGP used in the previous section to analyze the effects of outliers on the one-step-ahead forecast of the conditional covariance matrix. Because the PVC and the RPVC procedures have similar performance (see, Section 3.2 and Li et al. (2016)), hereafter we focus on the GPVC procedure and compare it with our robust proposal.

The h-steps-ahead forecast of the conditional covariance matrix can be obtained through

$$\hat{\Sigma}_y(h) = \hat{A}\hat{\Sigma}_{\hat{f}}(h)\hat{A}' + \hat{A}\hat{A}'\hat{\Sigma}_y\hat{B}\hat{B}' + \hat{B}\hat{B}'\hat{\Sigma}_y, \tag{9}$$

where $\hat{\Sigma}_{\hat{f}}(h)$ is h-steps-ahead prediction of the conditional covariance matrix of the estimated components $\hat{f}$, $\hat{\Sigma}_y$ is the estimated unconditional covariance matrix of y and $\hat{A}$ and $\hat{B}$ are estimated eigenvectors. Note that if $\hat{\Sigma}_y = I$, $\hat{\Sigma}_y(h) = \hat{A}\hat{\Sigma}_{\hat{f}}(h)\hat{A}' + \Sigma_\epsilon$ as presented in Li et al. (2016).

The one-step-ahead prediction of the volatility component is estimated using a Student-t quasi-maximum likelihood (QML) GARCH(1,1) model for the GPVC and by the robust procedure of Boudt et al. (2013) with the filter used in Trucíos et al. (2017) for the RPVC. Note that although using the RPVC we obtain robust estimates of matrices A and B used in (9), the volatility components evolving over time are still affected by outliers since they are the product of the columns of $\hat{A}$ multiply the original panel. To overcome this problem, we use the robust procedure of Boudt et al. (2013) to estimate the conditional covariance matrix of the volatility components.

Figures 2 and 3 report the MSE[7] and the MAE, respectively, for contaminated and

[7]Results for the MSE were cut-off in the value of 50 for small dimensions and in 3 for large dimensions to improve the visualization in the figure.

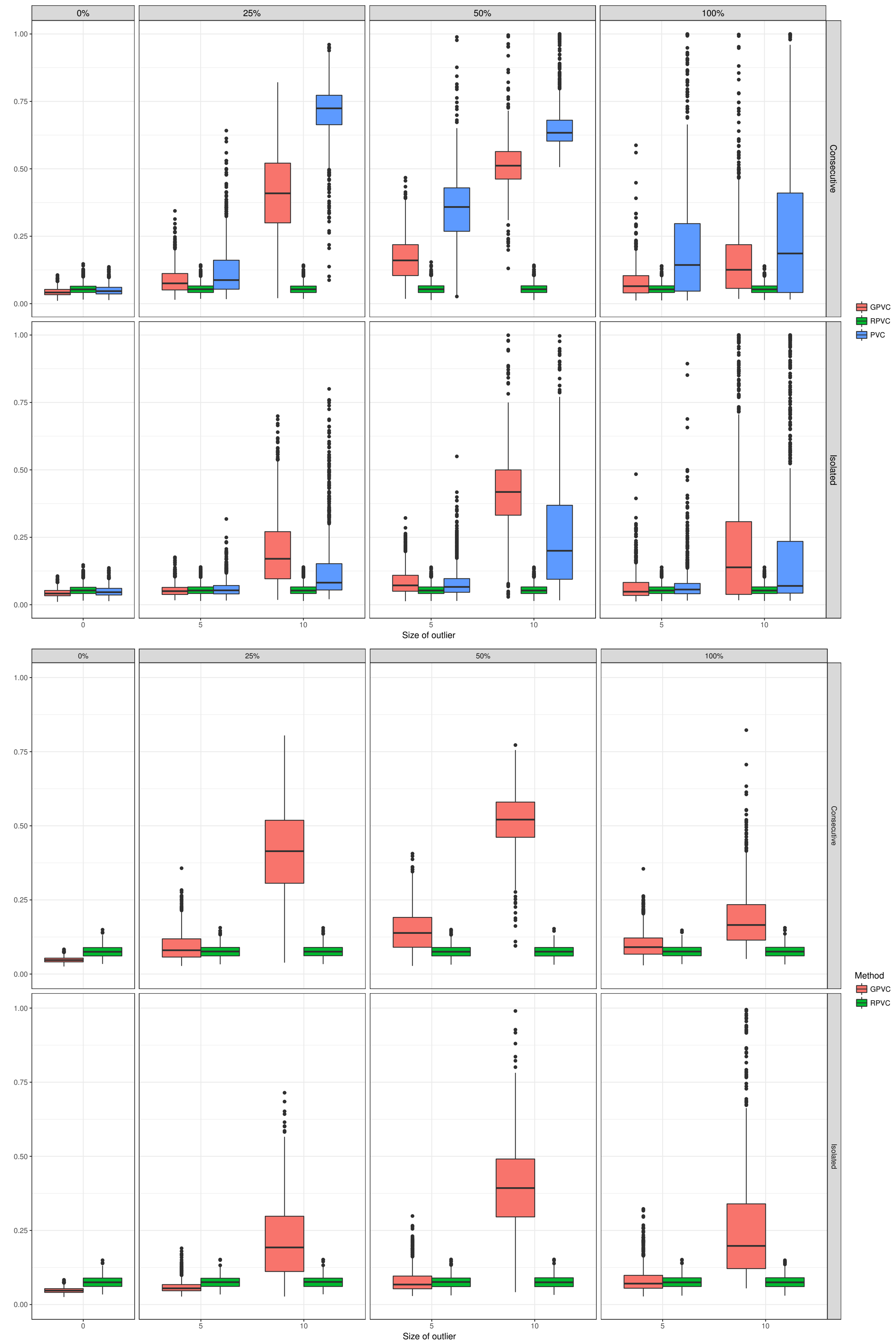

Figure 1: Boxplot of $d(\hat{\mathcal{M}}_1, \mathcal{M}_1)$ for uncontaminated (0%) and contaminated series with 25%, 50% and 100% of series contaminated. Dimension 8 (top panel) and 100 (bottom panel), $T = 1000$ and outliers of size $\omega = 0$, 5 and 10 standard deviations of the univariate uncontaminated process. 1000 replicates

uncontaminated series in small and large dimension cases[8]. The results show a devastating impact of outliers on the forecasting of the conditional covariance matrix when the non-robust procedure is used. Observe also that for uncontaminated series both procedures have a similar performance. However, when outliers are present in the series the advantage of the robust procedure is clear, even in small dimensions. Also note that for both criteria, when the dimension of the series is small and when only 25% of series are contaminated with outliers of size $\omega = 5$, the performance of both procedures are similar regardless if outliers are isolated or consecutive.

It is clear that the non-robust volatility components procedure is very sensitive to outliers and can lead to improper estimation and forecast of the conditional covariance matrix, even when the true number of volatility components with heteroskedastic dynamics is known. The consequences of using a non-robust procedure to forecast the conditional covariance matrix when outliers are present in the series can be disastrous leading for instance, to misspecified portfolio allocation and improper construction of risk measures.

To illustrate the effects of misspecification of the selected number of volatility components on the prediction of the conditional covariance matrix as well as to illustrate the effects on the portfolio allocation when using the predicted conditional covariance matrix. In this exercise, we consider a moderate dimension ($N = 40$). Table 3 reports the MSE, MAE, Frobenius distance and Eigenvalue loss function between the true one-step-ahead conditional covariance matrix ($\Sigma_{T+1|T}$) and the one-step-ahead forecast of the conditional covariance matrix ($\hat{\Sigma}_T(1)$) using the GPVC and RPVC procedures with one and two components. Table 3 also reports the annualized out-of-sample standard deviations of the true (weights are obtained using $\Sigma_{T+1|T}$) and selected (weights are obtained using $\Sigma_T(1)$) MVP returns.

Results in Table 3 are based on two DGP [9] and the first 25% of series were contaminated by additive outliers of size $\omega = 10$ at positions $t = 500, 501, 998$ and 999. Regardless of the numbers of selected volatility components, slightly better results in terms of MSE, MAE,

[8]The MSE and MAE are defined respectively as $\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}(\hat{\sigma}_{i,j}-\sigma_{i,j})^2}{N^2}$ and $\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}|\hat{\sigma}_{i,j}-\sigma_{i,j}|}{N^2}$ where $\hat{\sigma}_{i,j}$ and $\sigma_{i,j}$ are the elements of the predicted and true conditional covariance matrix.

[9]The DGP used are the factor model with one and two common factors. The common factor follows a Gaussian GARCH(1,1) process with parameters $\omega = 2$, $\alpha = 0.03$ and $\beta = 0.92$ for the factor model with one common factor (FM 1 Comp) and parameters $\omega = (1, 2)$, $\alpha = (0.07, 0.03)$ and $\beta = (0.83, 0.92)$ for the factor model with two common factors (FM 2 Comp).

Frobenius distance and Eigenvalue loss function are obtained using the GPVC procedure when there are not outliers in the series. In the presence of outliers, a clear superiority of the RPVC against the GPVC is observed.

Results in Table 3 also reveal that when the data is driven by one common factor, the GPVC procedure using one or two volatility components presents similar MSE, MAE, Frobenius distance and Eigenvalue loss function in the absence of outliers. Results using the RPVC procedure in the absence of outliers are also similar, with a slightly worse performance in terms of the Frobenius distance and the Eigenvalue loss function when two volatility components are used. In the presence of outlier, over-specification of the number of selected volatility components in the GPVC procedure leads to a worse performance while over-specification using the RPVC procedure improves the results.

For data driven by two common factors, underspecification of the number of selected volatility component using either GPVC or RPVC procedure presents a worse performance than using the correct number of volatility components when no outliers are present in the series. In the presence of outliers, underspecification of the number of volatility components leads to a worse performance when the RPVC procedure is used. For the GPVC procedure, a better performance is observed using GPVC with one volatility components than GPVC with two volatility components. However, results using GPVC with one or two volatility components are highly affected by outliers and none of them is superior to the results obtained using RPVC.

4 Empirical application

In this section, we implement the RPVC procedure to analyze the daily returns of stocks used in the construction of the Nasdaq-100 index traded from January 6, 2001, to May 12, 2017. Because not all stocks of the index were traded during the entire period, we ended up with $N = 73$ stocks. The daily prices are available at *finance.yahoo.com* and were downloaded on May 14, 2017, using the R package *quantmod* of Ryan (2017). Returns are computed as usual by $r_{i,t} = 100 \times log\,(P_{i,t}/P_{i,t-1})$, where $P_{i,t}$ denotes the adjusted closing price of the i-th stock at day t for $i = 1, ..., 73$. Following Engle et al. (2017), to avoid that very similar stocks being included in the analysis, we look for possible highly correlated

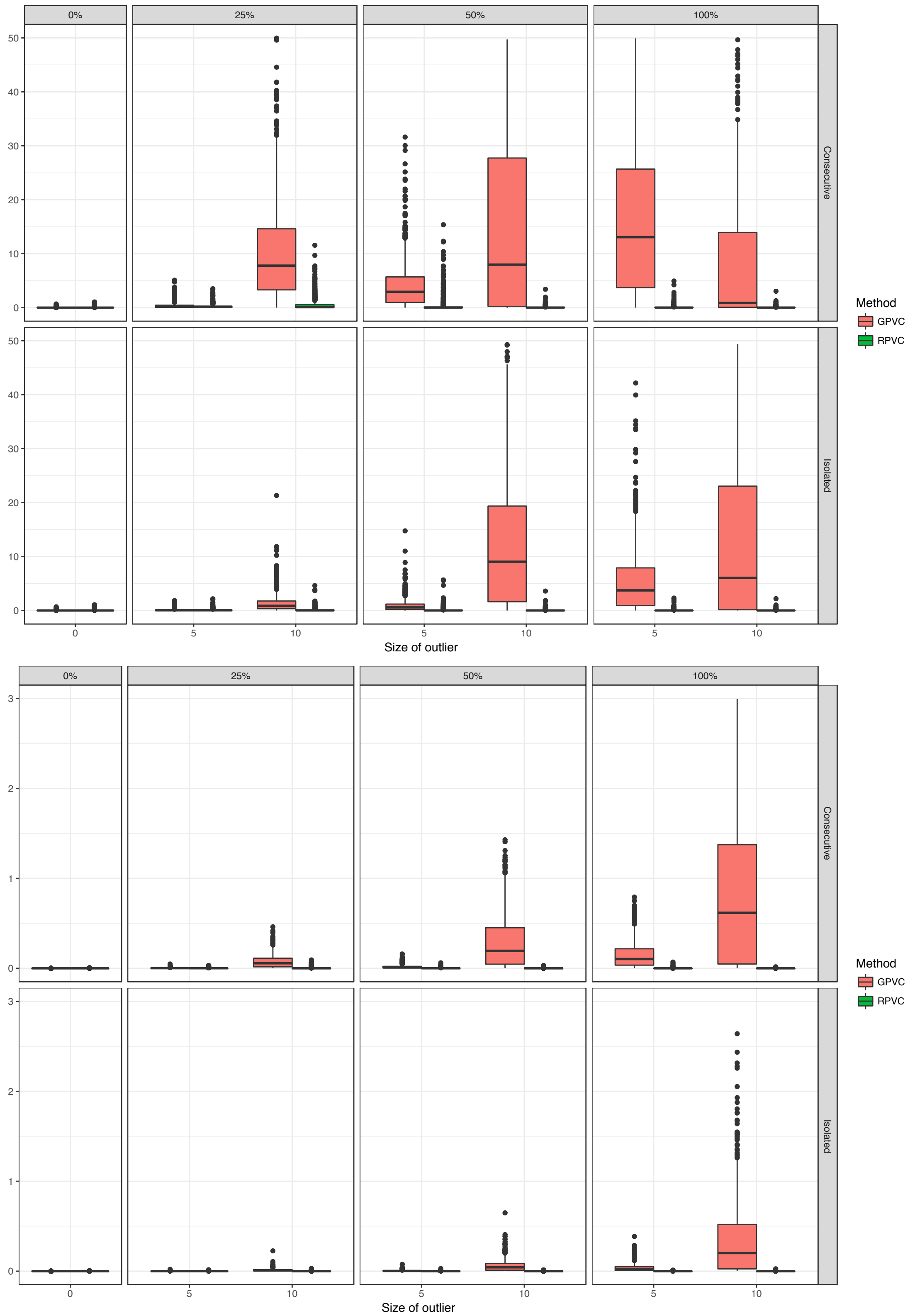

Figure 2: Boxplot of MSE $(\Sigma_{T+1|T}, \hat{\Sigma}_T(1))$ for uncontaminated (0%) and contaminated series with 25%, 50% and 100% of series contaminated. Dimension 8 (top panel) and 100 (bottom panel), $T = 1000$ and outliers of size $\omega = 0$, 5 and 10 standard deviations of the univariate uncontaminated process. 1000 replicates. Results were cut-off in the value of 50 for small dimensions and in 3 for large dimensions to improve the visualization in the figure.

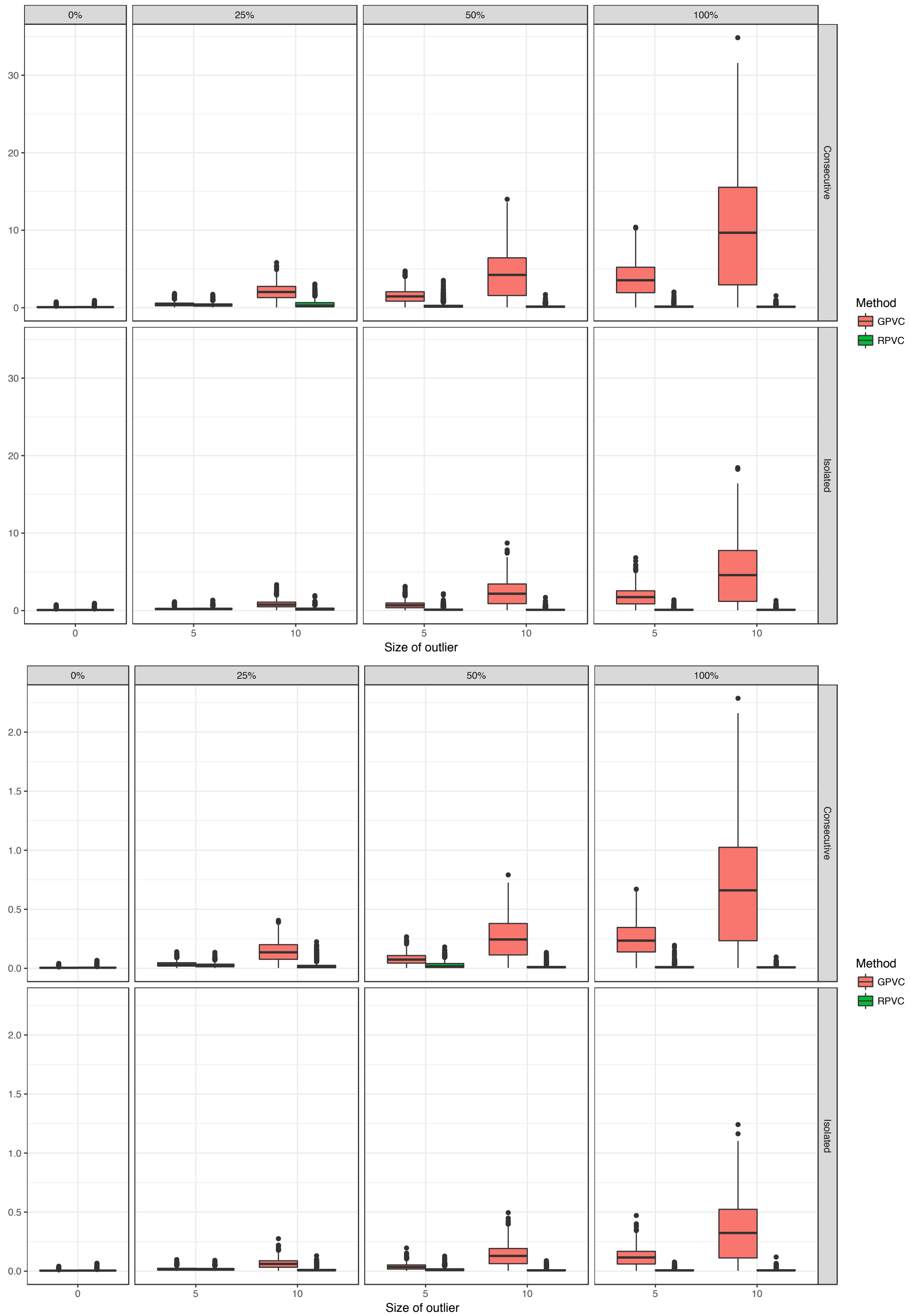

Figure 3: Boxplot of MAE $(\Sigma_{T+1|T}, \hat{\Sigma}_T(1))$ for uncontaminated (0%) and contaminated series with 25%, 50% and 100% of series contaminated. Dimension 8 (top panel) and 100 (bottom panel), $T = 1000$ and outliers of size $\omega = 0$, 5 and 10 standard deviations of the univariate uncontaminated process. 1000 replicates.

Table 3: Average MSE, MAE, Frobenius distance and Eigenvalue loss function between the true one-step-ahead conditional covariance matrix ($\Sigma_{T+1|T}$) and the one-step-ahead forecast of the conditional covariance matrix ($\hat{\Sigma}_T(1)$). Annualised standard deviation of the true (weights are obtained using $\Sigma_{T+1|T}$) and selected (weights are obtained using $\Sigma_T(1)$) out-of-sample MVP returns. Sample size $T = 1000$, dimension $N = 40$. Series uncontaminated by outliers ($\omega = 0$) and series with the first 25% of series contaminated by four outliers of size 10 standard deviation of the uncontaminated process ($\omega = 10$). Outliers positions $t = 500, 501, 998$ and 999. Monte Carlos replicates $M = 1000$. DGP: factor model with one and two common factors.

DGP	**Measure**	**One volatility component**				**Two volatility components**			
		GPVC		RPVC		GPVC		RPVC	
		$\omega = 0$	$\omega = 10$	$\omega = 0$	$\omega = 10$	$\omega = 0$	$\omega = 10$	$\omega = 0$	$\omega = 10$
FM 1 comp.	Avg. MSE ($\Sigma_{T+1\|T}, \hat{\Sigma}_T(1)$)	0.007	8.843	0.010	0.018	0.007	9.733	0.010	0.013
	Avg. MAE ($\Sigma_{T+1\|T}, \hat{\Sigma}_T(1)$)	0.047	0.787	0.055	0.072	0.047	0.933	0.055	0.064
	Avg. Frobenius ($\Sigma_{T+1\|T}, \hat{\Sigma}_T(1)$)	11.738	14149.485	16.160	29.147	11.738	15572.941	15.996	20.866
	Avg. Eigen ($\Sigma_{T+1\|T}, \hat{\Sigma}_T(1)$)	11.041	14143.931	15.456	28.444	11.041	15441.901	15.294	20.163
	Ann. SD of the true MVP	0.386							
	Ann. SD of the selected MVP	0.397	0.413	0.399	0.399	0.397	0.492	0.399	0.399
FM 2 comp.	Avg. MSE ($\Sigma_{T+1\|T}, \hat{\Sigma}_T(1)$)	0.013	1.848	0.015	0.074	0.011	2.969	0.013	0.038
	Avg. MAE ($\Sigma_{T+1\|T}, \hat{\Sigma}_T(1)$)	0.067	0.505	0.070	0.131	0.058	0.658	0.063	0.097
	Avg. Frobenius ($\Sigma_{T+1\|T}, \hat{\Sigma}_T(1)$)	21.217	2957.336	24.395	119.001	17.874	4750.618	21.203	61.000
	Avg. Eigen ($\Sigma_{T+1\|T}, \hat{\Sigma}_T(1)$)	18.021	2945.625	21.157	114.515	16.021	4583.397	19.173	56.567
	Ann. SD of the true MVP	0.404							
	Ann. SD of the selected MVP	0.415	0.423	0.420	0.418	0.414	0.525	0.426	0.422

(larger than 0.95) pairs of stocks and if any, we remove one of them of our analysis. Figure 4 reports the unconditional correlation among the 73 stocks where is observed that none of them is larger than 0.83. The maximum correlation between pairs is 0.808 and more of correlations (87.7%) are in the interval $[0.2, 0.5]$, 8.7% are smaller than 0.2 and 3.5% larger than 0.5.

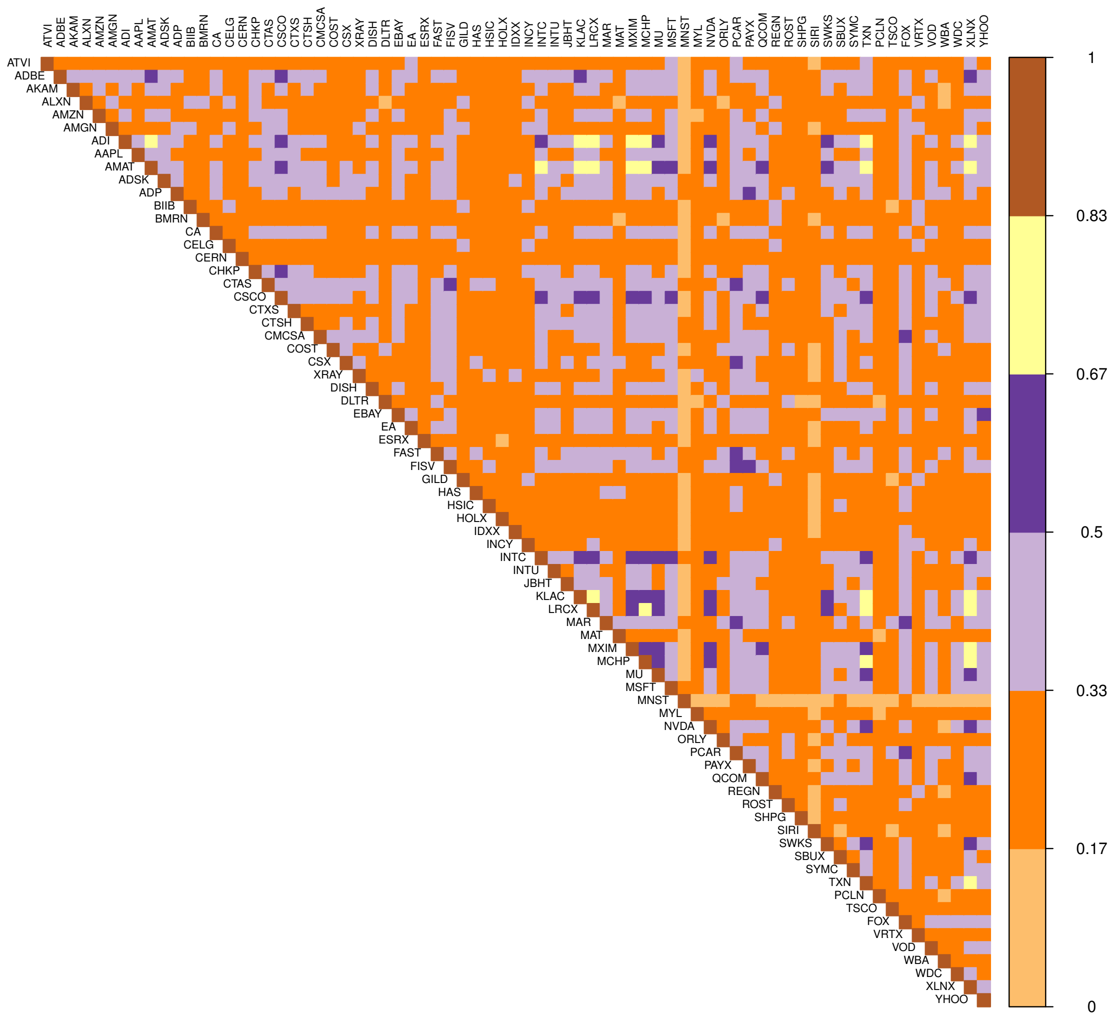

Figure 4: Unconditional correlation among stocks.

With illustrative purposes we use the one-step-ahead forecast of the conditional covariance matrix to estimate the 1% and 5% Value-at-Risk (VaR) of the equal-weight portfolio as well as to construct the minimum variance portfolio (MVP) with short-sale constraint. The VaR is calculated assuming a Student-t distribution where the degrees of freedom is estimated by maximum likelihood using the devolatilized residuals[10] and the MVP are rebalanced daily.

In the VaR and the MVP applications, we use a rolling window scheme with window size of 1000 days and all results are compared with the GPVC procedure, the OGARCH (Alexander and Chibumba, 1996) model, the classical Risk Metrics (RM) methodology (Morgan, 1996), the new version called Risk Metrics 2006 (RM2006) (Zumbach, 2007), the DECO model (Engle and Kelly, 2012) and with the DCC model using composite likelihood (Pakel et al., 2014). We use the OGARCH procedure because this method has been used in several papers as a dimension reduction technique benchmark (Santos and Moura, 2014; Becker et al., 2015; Santos and Ferreira, 2017). The OGARCH model was estimated as in Becker et al. (2015), which means, considering the number of components equal to the number of series and each component is modelled as a GARCH(1,1) process. The GARCH model estimated by quasi-maximum likelihood with Stunden-t distribution were considered in all cases, except when the RPVC procedure was implemented, in which case was used the robust procedure of Boudt et al. (2013).

The number of selected volatility components in the GPVC and RPVC procedures could be estimated using the ratio estimator (Lam and Yao, 2012; Ahn and Horenstein, 2013), the BN (Bai and Ng, 2002) and the Kaiser-Guttman (Guttman, 1954) criteria at each window. Because it would be computationally cumbersome, and mainly because there is no agreement of each method is the best, we used the three criterion to the complete series. The ratio estimator criterion suggests using one component in both cases (GPVC and RPVC), the BN criterion suggests using three components in both cases, while the Kaiser-Guttman criterion suggests using three components when the GPVC procedure is used and four components in the robust procedure. Given these results in all the windows, we apply the GPVC and RPVC procedures using one, two, three and four volatility components. The conditional variance of the volatility component is forecasted using the same strategy described in Subsection 3.3 and conditional covariance matrices in cases with more than one volatility component

[10] For $t = 1, ...1000$, the devolatilised residuals were obtained through $e_{p,t} = r_{p,t}/\sqrt{\hat{\sigma}^2_{p,t}}$ where $r_{p,t} = \omega \times (r_{1,t}, ...r_{73,t})'$ and $\hat{\sigma}^2_{p,t} = \omega \hat{H}_t \omega'$ are the portfolio returns and variances at time t respectively with ω and $\hat{H}_t$ being a vector of equal-weights and the estimated conditional covariance matrix respectively. The VaR for the period $(T, T+1)$ is obtained by $VaR = T_{\nu,\alpha}\hat{\sigma}_{p,T+1|T}$ where $T_{\nu,\alpha}$ is the α quantile of the standardised Student-t distribution with ν degrees of freedom (ν is estimated using the portfolio innovations) and $\hat{\sigma}^2_{p,T+1|T} = \omega \hat{H}_{T+1|T}\omega'$ is the one-step-ahead forecast conditional portfolio variance.

are forecasted using the DCC model. The DCC model was estimated by Aielli (2013) and Boudt et al. (2013) methods for the GPVC and RPVC procedures respectively.

Table 4 reports the backtesting performance of different methods when estimating the 1% and 5% VaR of the equal-weight portfolio. The percentage of violation returns smaller than the VaR, is presented in the first column and the p-values of the back-testing tests of unconditional coverage (UC) (Kupiec, 1995), independence (IND) and conditional coverage (CC) (Christoffersen, 1998), are given in the next three columns. The table also reports the results for the dynamic quantile (DQ)(Engle and Manganelli, 2004) and VaR quantile tests (Gaglianone et al., 2011). The estimated expected shortfall (ES) with its p-value of the back-testing test (ESTest) of McNeil and Frey (2000) for the null hypothesis that the ES is estimated correctly by the model is also reported.

The percentage of violations using the dimension reduction techniques (GPVC, RPVC, OGARCH) is close to the nominal one and in those cases, the unconditional coverage (UC) and conditional coverage (CC) tests fail to reject the null hypotheses at 5% of significance level. Note that results for the 5% VaR using the GPVC procedure are similar regardless of the number of selected volatility components used. However, the CC test rejects the null hypothesis of independence in the estimation of the 5% VaR for all GPVC procedures (one to four volatility components), with no rejection for the RGPC procedures, while for the 1% VaR the null hypothesis of independence is rejected at 5% level for the RPVC procedure with one and two volatility components. The percentage of violations using RM2006 methodology is slightly smaller than the nominal value, with the UC and CC tests failing to reject the null hypotheses. The RM and RM2006 methodologies have a good performance for the 5% VaR, but for the 1% VaR, estimated by RM methodology, the null hypothesis is rejected at 5% level by the UC and CC tests. The DCC and DECO models do not have a good performance according to the IND test for the 1% VaR. The DQ test rejects the null hypothesis for the 1% VaR, at 5% level, in the DCC and DECO models, RM methodology and RPVC with one and two components, while the VQR test rejects the null hypothesis for the 1% VaR estimated by the GPVC, RM and RM2006 procedures and OGARCH model at 10% level. The McNeil and Frey (2000) test for the ES does not rejects the null hypothesis at 5% level in any test, and at 10% level rejects the null hypothesis when using RPVC procedure

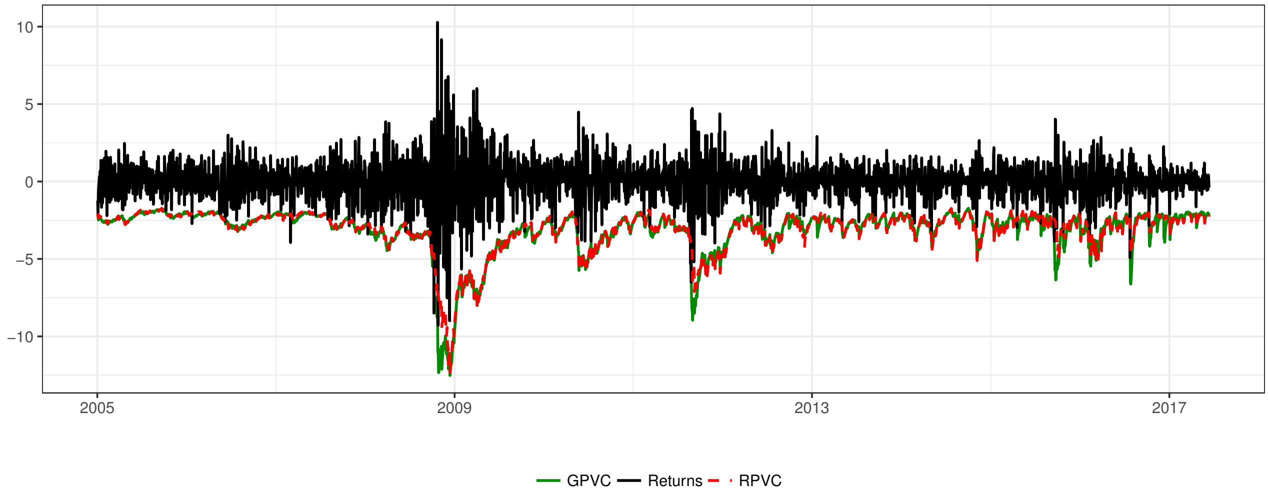

Figure 5: 1% VaR of the equal-weight portfolio using the GPVC (solid green line) and RPVC (dashed red line) procedures (considering one volatility component)

with one and two components, for the 1% VaR and for the 5% VaR using RPVC model with two components. Considering that when applying the RPVC procedure it is better to overestimate the number of components than underestimate, we would probably not use one or two volatility components, but three or four, for most of the windows, we could say that in terms of VaR violations the performance of the RPVC procedure is quite good in relationship to all compared methods, especially to the GPVC procedure. We will comment about the selection of the number of components later.

Figure 5 shows the 1% VaR of the equal-weight portfolio using the GPVC and RPVC procedures with one volatility component. Note that after large returns, the VaR obtained using the GPVC (solid green line) procedure is unnecessarily larger than the obtained using the RPVC (dashed red line) procedure, implying in more capital requirements. Additional figures comparing the RPVC with other procedures are in the supplementary material.

We now analyze the performance of the selected MVP according to economic criteria. The results are presented from January 3, 2005, to May 12, 2017 (entire out-of-sample period) and also for August 1, 2007, to December 31, 2013 (high volatile out-of-sample period). Following Engle et al. (2017), Gambacciani and Paolella (2017) and Trucíos et al. (2018), Table 5 reports three annualized performance measures based on the observed returns of the selected portfolios: standard deviation (SD) and information ratio (IR) and Sortino ratio (SO) (Sortino and van der Meer, 1991). For the sake of comparison, we also implement the

Table 4: Performance of different methods when estimating the 1% and 5% VaR of the equal-weight portfolio given by backtesting procedure. Percentage of violations (returns smaller than VaR) and p-values of the unconditional coverage (UC), independence (IND), conditional coverage (CC), dynamic quantile (DQ), VaR quantile regression (VQR) tests. Estimated expected shortfall (ES) and p-values that ES is estimated correctly by the model. VaR 1% (top panel) and VaR 5% (bottom panel). In bold, the cases where the p-values are smaller than 0.05.

	Method	% violations	UC	IND	CC	DQ	VQR	ES	ESTest
1% VaR	GPVC 1VC	0.868	0.448	0.492	0.592	0.668	0.053	-3.817	0.230
	RPVC 1VC	0.932	0.699	**0.029**	0.086	**0.020**	0.446	-3.787	0.091
	GPVC 2VC	0.835	0.342	0.508	0.512	0.612	0.057	-3.829	0.182
	RPVC 2VC	0.996	0.983	**0.039**	0.118	**0.023**	0.542	-3.801	0.089
	GPVC 3VC	0.835	0.342	0.508	0.512	0.611	0.055	-3.825	0.155
	RPVC 3VC	0.964	0.839	0.445	0.731	0.506	0.266	-3.781	0.210
	GPVC 4VC	0.900	0.567	0.476	0.659	0.652	0.072	-3.822	0.244
	RPVC 4VC	1.060	0.737	0.400	0.664	0.644	0.275	-3.794	0.205
	OGARCH	0.867	0.448	0.492	0.592	0.553	0.089	-3.814	0.189
	RM	1.478	**0.012**	0.186	**0.018**	**0.000**	0.051	-3.853	0.352
	RM 2006	0.803	0.254	0.197	0.227	0.215	0.080	-3.956	0.351
	DCC	0.964	0.839	**0.034**	0.103	**0.024**	0.950	-3.942	0.458
	DECO	1.028	0.875	**0.044**	0.130	**0.034**	0.811	-3.911	0.575
5% VaR	GPVC 1VC	4.724	0.475	**0.022**	0.056	0.254	0.881	-2.761	0.368
	RPVC 1VC	5.141	0.719	0.204	0.418	0.729	0.881	-2.701	0.162
	GPVC 2VC	4.724	0.475	**0.022**	0.056	0.251	0.851	-2.767	0.430
	RPVC 2VC	5.077	0.844	0.230	0.477	0.849	0.881	-2.711	0.088
	GPVC 3VC	4.724	0.475	**0.022**	0.056	0.253	0.883	-2.764	0.403
	RPVC 3VC	5.109	0.780	0.412	0.687	0.925	0.881	-2.700	0.143
	GPVC 4VC	4.724	0.475	**0.022**	0.056	0.253	0.881	-2.762	0.391
	RPVC 4VC	5.206	0.601	0.592	0.756	0.954	0.881	-2.709	0.114
	OGARCH	4.916	0.830	0.141	0.331	0.507	0.947	-2.755	0.455
	RM	4.948	0.895	0.132	0.319	0.621	0.918	-2.747	0.132
	RM 2006	4.434	0.140	0.146	0.117	0.333	0.286	-2.837	0.429
	DCC	4.692	0.425	0.437	0.538	0.682	0.353	-2.806	0.146
	DECO	4.788	0.585	0.377	0.583	0.529	0.478	-2.796	0.234

equal-weighted (EW) portfolio.[11]. Note that, as mentioned by Engle et al. (2017) and Ledoit and Wolf (2017), notwithstanding high information and Sortino ratios as well as minimum standard deviation are all desirable properties, the MVP is calculated to achieve minimum variance and SD should be the main focus in the comparison.

First, we analyze the results in terms of SD, the most important criterion. Regardless of the number of volatility components used, the RPVC procedure always outperforms the GPVC. Additionally, in most of the cases, the RPVC method outperform the GPVC in

[11]See, Fan et al. (2012), Engle et al. (2017) and Gambacciani and Paolella (2017) for some references where the naive equal-weighted portfolio has also used with comparison purpose.

terms of information and Sortino ratios. The superiority of the RPVC procedure over the GPVC procedure is in concordance with the Monte Carlo experiments carried out in Section 3. So, in the following, we only compare the RPVC method with the other methods. In terms of SD, the DECO model never outperform the RPVC method, the OGARCH model only outperforms the RPVC method with one component in the entire period, and the RM methodology only outperforms the RPVC method with one and two components in the second period. However, the DCC and RM2006 models outperform the RPVC method in all cases.

In terms of the information and Sortino ratios, in general, the RM methodology present the best performance in both periods, followed by the RM2006 methodology and the DCC models, and all three methods present better performance than the RPVC method, regardless of the number of components used. The DECO model is outperformed by the RPVC method in all cases, except in the first period when RPVC is applied with three components, while The DECO model is outperformed by the RPVC method in all cases, except in the first period when RPVC is applied with three and four components. Similar results where dimension reduction techniques are outperformed by multivariate volatility models such as EWMA, ORE (Foster and Nelson, 1996) and RiskMetrics in a MVP context can be found in, for instance, Han (2006), Santos and Moura (2014), Becker et al. (2015), Caldeira et al. (2017), Santos and Ferreira (2017) and Ledoit and Wolf (2017). Han (2006) and Han (2007) point out that not necessarily better statistical models have a better portfolio performance than simplest ones. However, the RM2006 methodology and the DCC model which outperformed the RPVC in terms of SD, IR and SO had poor performance in estimating the 1% VaR of the EW portfolios. So, in general, the RPVC produced good results.

Additional Monte Carlo experiments (see supplementary material) report that in the presence of outliers a better performance in the estimation of the one-step-ahead conditional covariance matrix[12] do not necessarily lead to a smaller annualized out-of-sample standard deviation of the selected MVP returns. In particular, results show that considering the DCC model as DGP, the RPVC procedure outperforms the DCC procedure in terms of estimation of the one-step-ahead conditional covariance matrix. However, the estimation

[12]Better performance in terms of MSE, MAE, Frobenius distance and distance between the largest eigenvalues of the true and estimated covariances matrices.

using the DCC leads to a small annualized out-of-sample standard deviation of the selected MVP returns. These results could explain the small SD reported in Table 5 when using non-robust procedures such as RM2006 and DCC.

Table 5: Annualised performance measures for the equal-weight portfolio and the selected MVP using GPVC and RPVC procedures (with one to four volatility components) and other methods. In bold the best performance for each criterion

	Jan 3, 2005 - May 12, 2017			Aug 1, 2007 - Dic 31, 2013		
Method	SD	IR	SO	SD	IR	SO
1/N	20.7347	0.6272	0.8796	24.8567	0.5064	0.7083
GPVC 1VC	16.8537	0.6773	0.9445	20.2005	0.6159	0.8535
RPVC 1VC	16.8393	0.7875	1.0998	19.9686	0.6419	0.8856
GPVC 2VC	16.8171	0.6888	0.9619	20.1418	0.6400	0.8885
RPVC 2VC	16.7705	0.7924	1.1064	19.9367	0.6437	0.8881
GPVC 3VC	16.8266	0.7105	0.9904	20.1810	0.6431	0.8901
RPVC 3VC	16.6758	0.6939	0.9672	19.9008	0.6065	0.8385
GPVC 4VC	16.7954	0.7131	0.9930	20.1550	0.6505	0.8989
RPVC 4VC	16.6562	0.7283	1.0194	19.8266	0.6757	0.9415
OGARCH	16.8062	0.7801	1.0935	20.1997	0.5559	0.7699
RM	16.8517	**1.0880**	**1.5548**	19.9113	**0.9218**	**1.2948**
RM 2006	**16.3270**	0.9890	1.3873	**19.4100**	0.7997	1.1005
	16.3474	0.9307	1.3059	19.5645	0.7779	1.0735
DECO	16.8648	0.7086	0.9892	20.1982	0.5893	0.8137
DCC	16.3569	0.9204	1.2918	19.5649	0.7769	1.0733

(a) SD: Standard deviation of the out-of-sample portfolio returns multiplied by $\sqrt{252}$. (b) IR: Annualised information ratio. (c) SO: Annualised Sortino ratio.

Figure 6 plots the square of the estimated volatility components and the square of their conditional volatility for the first four components obtained using GPVC and RPVC methods using the entire period. We can observe that all components have volatility clustering and that the first and fourth components, estimated by both methods are very similar, while the second component estimated by the GPVC method is similar to the third component estimated by the RPVC method, while the third component estimated by the GPVC method is similar to the second component estimated by the RPVC method. Thus, we should compare the estimated first, second, third and fourth volatility components estimated by the GPVC method, respectively with the first, third, second and fourth volatility components estimated by the GPVC method. The main difference is in the estimated volatility of the second and third volatility components, with lower variability, as expected, in the robust RPVC method.

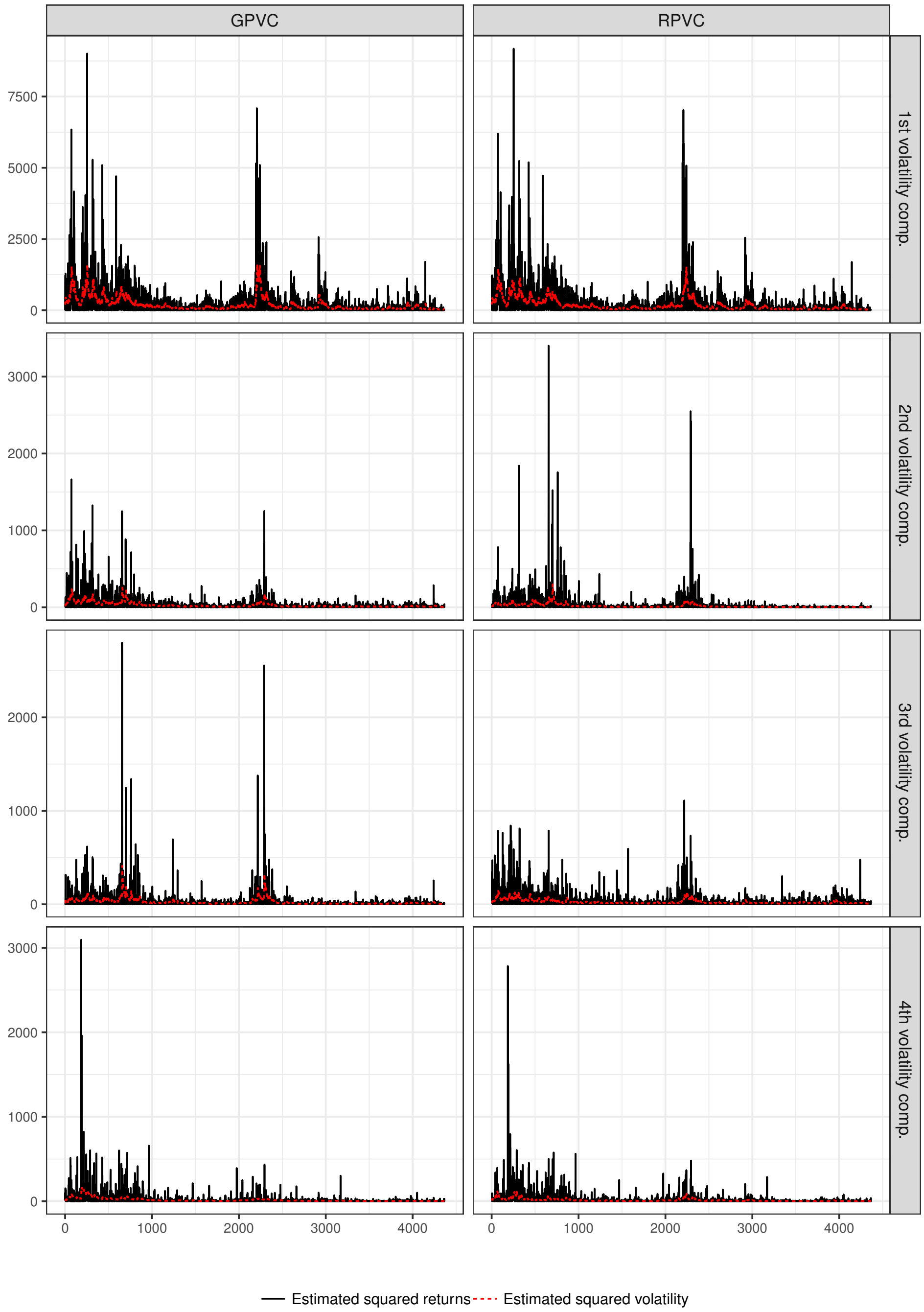

Figure 6: Plot of the square of the estimated volatility components (solid black line) and the square of their conditional volatility for the first four components obtained using GPVC and RPVC methods (dashed red line) using the entire period.

5 Conclusions and further research topics

In this paper, we focus on the principal volatility components procedure of Hu and Tsay (2014a) and Li et al. (2016). These procedures extract few components with time-varying volatility and the remaining components with constant volatility tackling the problem of the curse of the dimensionality.

We analyze the problem of modelling and forecasting the conditional covariance matrix via principal volatility components in the presence of outliers and show that just a few outliers are sufficient to affect drastically the volatility components and the estimation of the conditional covariance matrix.

To estimate the number of selected volatility components we used the estimator criterion (Lam and Yao, 2012; Ahn and Horenstein, 2013), the BN criterion (Bai and Ng, 2002) and the Kaiser-Guttman criterion (Guttman, 1954). The use of the ration estimator and BN criteria estimated the number of components close (or equal) to the true number of factors. However, the Kaiser-Guttman criterion reports problems to estimate correctly the number of components when the ratio factors/dimension increase, for that reason we do not recommend its use in such cases.

A new and robust procedure with good finite sample properties based on a robust estimator of the unconditional covariance matrix, a weighted estimator and robust filters were introduced.

The principal volatility components approach brings an innovative contribution to the field of modelling and forecasting multivariate volatility, managing portfolios and quantifying risk. However, it is necessary to caution when the data is contaminated by outliers because disastrous results can be obtained when using the non-robust procedures of Hu and Tsay (2014a) or Li et al. (2016). This paper contributes to the literature in two ways: it call the attention to the risk of using these procedures in the presence of outliers and introduces an approach robust to outliers and with a similar performance in uncontaminated series.

In our empirical application, the one-step-ahead forecast of conditional covariance matrix was used to estimate the VaR and also to construct the MVP. In both applications, the RPVC procedure had a better performance than the GPVC. This results are in concordance with our Monte Carlo experiments and show the superiority of the RPVC procedure against the GPVC one.

The problem of dealing with $N/T \to 1$ or even $N > T$ has not been addressed here. This topic, as well as PVC in switching regime, are in our research agenda.

The aim of this paper is not to compare the predictability of the volatility using different approaches but to robustify the principal volatility components method. An extensive comparison using other recently procedures such as Matteson and Tsay (2011), Matilainen et al. (2015), Peña and Yohai (2016), Barigozzi and Hallin (2017) among other in different scenarios is an interesting further research topic.

Because there are a large number of methods to model volatility and none clearly dominate the others, as suggested by the reviewer, it may be helpful to consider forecasting combination method. Combination forecasting could also come in hand to tackle to problem of selecting the "right" number of volatility components in the (robust) principal volatility components method.

Finally, some papers such as Han (2006), Santos and Moura (2014), Becker et al. (2015), Caldeira et al. (2017), Santos and Ferreira (2017) and Ledoit and Wolf (2017) have reported in their empirical application that in a MVP context, dimension reduction techniques are sometimes outperformed by alternative multivariate volatility models such as EWMA, ORE (Foster and Nelson, 1996) and RiskMetrics. An interesting research topic is to evaluate some recent dimension reduction techniques and compare it in an MVP context with other multivariate volatility models as well as analyze the reasons why a better performance in alternative models is observed in the papers previously mentioned.

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